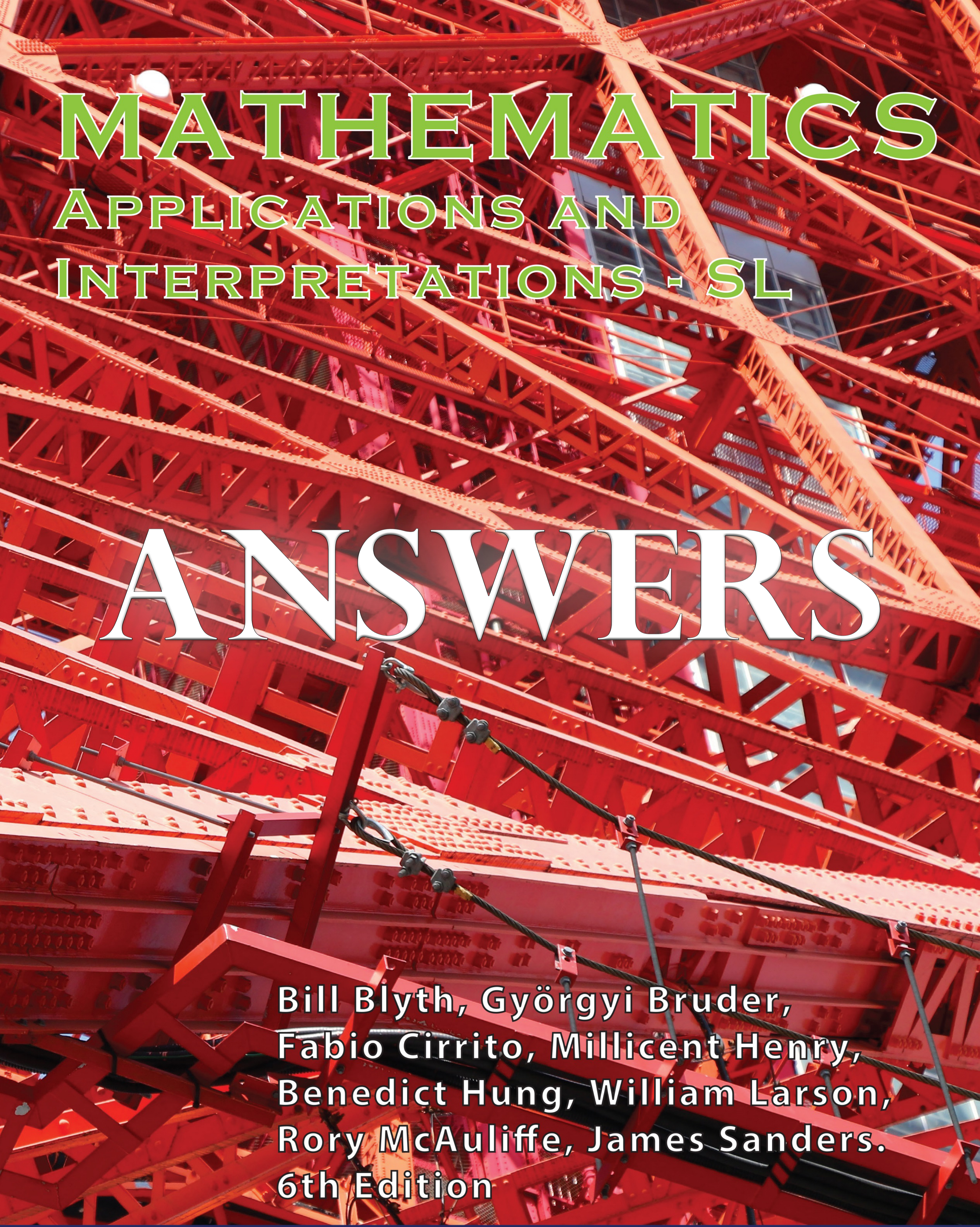

MATHEMATICS

Applications and Interpretations - SL

ANSWERS

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6th Edition

For use with the I.B. Diploma Programme

Exercise A.4.1

1.	a	1250 km	b	0.0200 gm	c	50100 sec		
	d	4520 mm	e	0.0300 m	f	4520 kg	g	0.00173 sec.
2.	a	0.2%	b	0.16%	c	0.1%		
	d	0.1%	e	16.7%	f	12.5%	g	50%

3. 0.04%

4. 0.01%

5. 3×10^{-9} %(!)

Exercise A.4.2

1. a ±0.05 cm b 0.005cm c 0.0005 cm - these are approximate!

2. i 14.577 to 14.828 cm ii 16.909 to 17.497 cm^2.

3. ~0.1%

4. i 254.34 to 260.16 m^2 ii 379.16 to 394.57 m^3.

5. These answers all depend to some extent on the values of *a* & *b*.
   i ~15% ii usually considerably larger than the data.
   iii ~20% iv usually considerably larger than the data.

6. Yes to all of these!

7. 0.005×10^{24}

8. 12%

9. 7%

Exercise A.5.1

1. a $54 b $57.30 c $59.00
2. a
3. $2 697.70
4. 12.7%
5. Over 100%
6. 9%
7. Approx. £31 500 000

Exercise A.5.2

1. a $54 750 b $310 250 c $1 658.58 (payment at beginning of month).
2. A yields $7 985 and B yields $7 668. A is best.
3. a $302.13 b $3 127.80
4. a $8 418 b $6 459 c $5 528 Interest $581 220
5. $5 837
6. a $24 952 b 18 321
7. Option A is 8%, option B 8.5%, option C 7.5%. Option C has the lowest rate but will result in the largest total interest.
8. a $525.74 b $300
9. Option A $31 186, option B $35 072, option C $35 094 - option C is best.

Exercise A.6.1

1 a $x = 1, y = 2$ b $x = 3, y = 5$ c $x = -1, y = 2$
d $x = 0, y = 1$ e $x = -2, y = -3$ f $x = -5, y = 1$

2 a $x = \frac{13}{11}, y = \frac{17}{11}$ b $x = \frac{9}{14}, y = \frac{3}{14}$ c $x = 0, y = 0$
d $x = \frac{4}{17}, y = -\frac{22}{17}$ e $x = -\frac{16}{7}, y = \frac{78}{7}$ f $x = \frac{5}{42}, y = -\frac{3}{28}$

3 a -3 b -5 c -1.5

4 a $m = 2, a = 8$ b $m = 10, a = 24$ c $m = -6, a = 9$.

5 a $x = 1, y = a-b$ b $x = -1, y = a+b$ c $x = \frac{1}{a}, y = 0$
d $x = b, y = 0$ e $x = \frac{a-b}{a+b}, y = \frac{a-b}{a+b}$ f $x = a, y = b-a^2$

Exercise A.6.2

1 a $x = 4, y = -5, z = 1$ b $x = 0, y = 4, z = -2$
c $x = 10, y = -7, z = 2$ d $x = 1, y = 2, z = -2$
e $\varnothing$ f $x = 2t-1, y = t, z = t$
g $x = 2, y = -1, z = 0$ h $\varnothing$

Exercise A.6.3

a $x = -0.618, y = 3.62$ & $x = 1.62, y = 1.38$ b No solutions
c $x = -0.851, y = 1.30$ & $x = 2.35, y = 7.71$ d $x = -4.78, y = 19.6$ & $x = 1.78, y = 6.44$
e $x = 1, y = 6$ f $x = -1.17, y = -2.47$

Exercise A.6.4

1. Pencil \$0.25 & biro \$1.10
2. Widget 1.3 gm & grommet 2.7 gm.
3. Bikkieflakes: 90 gm & Crunchybix: 150 gm.
4. Shirt \$30, jacket \$80.
5. 15 & 16.
6. 405 cm^3.
7. a $y = 3x^2 - 2x + 4$ b 44.
8. 1.47 hrs.
9. A: 1.2, B: 0.9, C: 1.4
10. $S = 3x + 5y + 2z$ $S(2,3,4) = 29$
11. a $y = x^2 - 5x - 7, y = 8 - 2x^2$ b (–1.55,3.18) & (3,22,–12.73)
12. 11, 12, 13.
13. 17,24 & 38.

Exercise B.4.1

1. a $y = 3x + 2$ b $y = 2.4x + 1.3$ c $y = 1.6x - 1.2$

   d $y = -0.9x + 2.3$ e $y = -0.3x + 2.6$ f $y = 2.1x + 3.7$

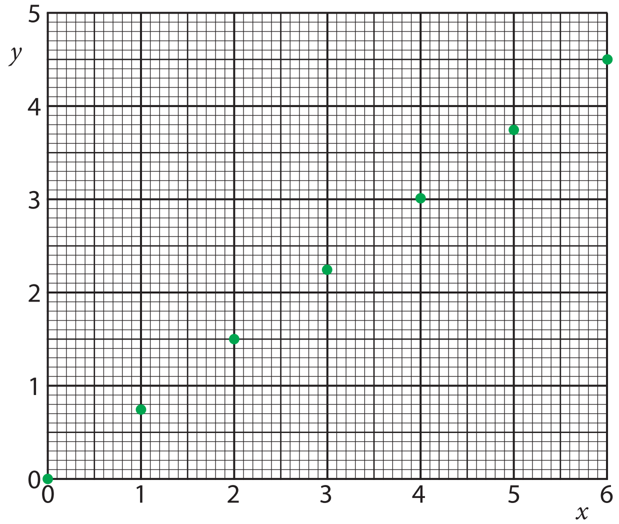
   g $y = 0.1x + 0.1$

2. Cost = 0.7956 × number of fliers + 25

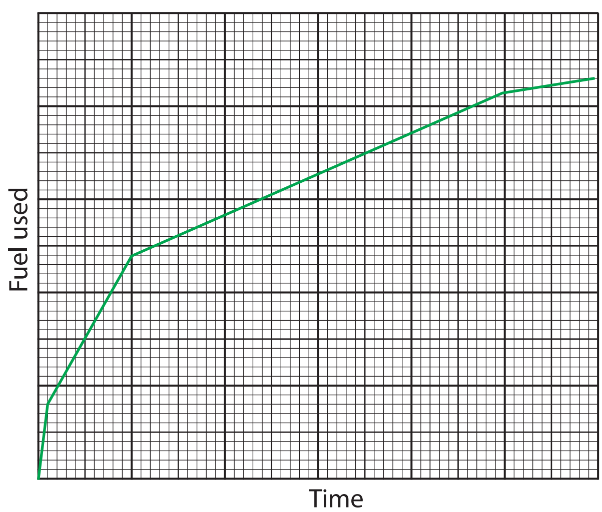
3. ~$43 assuming the wait time is also proportional to the distance.

4. $B = 1.35F + 0.5$

Exercise B.4.2

1. 

2. $$y = \begin{cases} 2x, 0 \le x < 1 \\ 0.5x, 1 \le x < 3 \\ x, 3 \le x \le 5 \end{cases}$$

3. 

4. $y=\begin{cases} 1, 0\le x<2 \\ x, 2\le x<4 \\ 13-2x, 4\le x<6 \end{cases}$

5. a $12 b $110 c Profit of $180.

6. $C=\begin{Bmatrix} 15n, 0\le n\le 100 \\ 12n+300, 100<n\le 500 \\ 10n+1300, 500<n\le 1000 \end{Bmatrix}$

7. $A = 4$, $B = -10$.

8. Number of ways $= n!, n\in\mathbb{Z}^+$ it is the domain that is most important in specifying this discrete function.

Exercise B.4.3

1. a $y=2x^2+x+3$ b $y=-x^2+3x+1$ c $y=3x^2+4x+2$

   d $y=3x^2-2x+1$ e $y=-1.1x^2-0.4x+1.5$ f $y=2.3x^2+1.1x-4.6$

2. Not a unique answer: $a\approx 2.9$, $b\approx 1.2$, $c\approx 2$.

3. It is not symmetric.

4. a (–10,0), (0,45), (10,0) b $y=45-0.45x^2$

5. $y=1.5x^2-3x+3$

x	0.20	0.60	1.00	1.40	1.80	2.20
y	2.27	1.76	1.79	1.88	2.44	3.55
Model	2.46	1.74	1.50	1.74	2.46	3.66
Error	-0.19	0.02	0.29	0.14	-0.02	-0.11

6. The errors are quite large. A model is: $y=4x^2+2x+3$. The error table is:

x	-2.00	-1.00	0.00	1.00	2.00	3.00
y	10.98	4.54	0.90	12.32	20.27	49.53
Model	15.00	5.00	3.00	9.00	23.00	45.00
Error	-4.02	-0.46	-2.10	3.32	-2.73	4.53

7. $y=x^2-2x+1$

x	-2.00	-1.00	0.00	1.00	2.00	3.00
y	9.24	4.18	0.00	-0.77	1.27	4.85
Model	9.00	4.00	1.00	0.00	1.00	4.00
Error	0.24	0.18	-1.00	-0.77	0.27	0.85

8. Stone arch bridges are usually circular.
The gothic arch is two intersecting circular arcs.
Reflector dishes are reputed to be rotated parabolas.

Exercise B.4.4

1. a $y=1\times 2^x$ b $y=3\times 5^x$ c $y=1.4\times 2.6^x$

d $y=2.7\times 2.5^x$ e $y=35\times 0.6^x$ f $y=0.2\times 3.1^x$

2. $2 023.

3. a $P(t)=380e^{0.01824t}$ b 2 355 c 91 hours

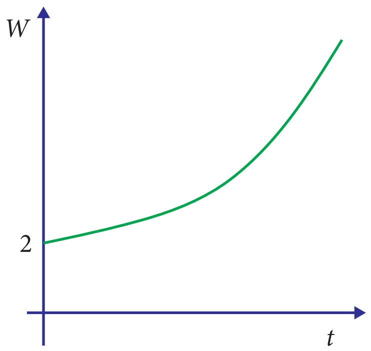
d The population will not continue to rise indefinitely due to limited food supplies. The actual limits are unclear.

4. Working in millions: $P(n)=9.98\times 0.674^n$ with an r value of –0.97 (good fit).
Or $P(n)=9.98e^{-0.394n}$ with an r value of –0.97 (good fit).

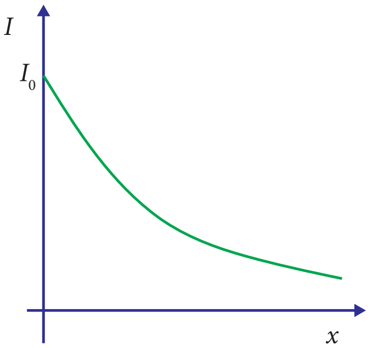
5. a $T=458e^{-0.824n}$ b 1.85 hrs.

6. 35 years.

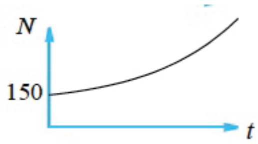
7. a 0.0013 b 2.061 kg c 231.56 yrs
d



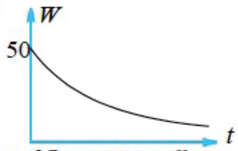
8. a 0.01398 b 52.53% c 51.53 m d 21.53 m
e



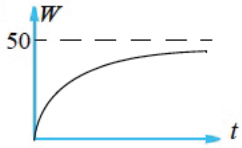
9. a i 157 ii 165 iii 191 b 14.2 yrs c 20.1 yrs d



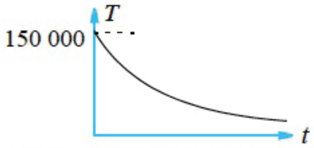
10. a 50 b 0.0222 c 17.99 kg d



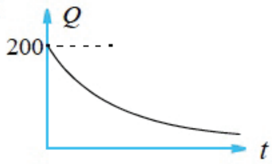
e



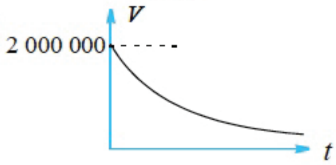
11. a 15 000°C b i 11 900°C ii 1500°C c 3.01 million yrs
d



12. a 0.0151 b 12.50gm c 20 years d

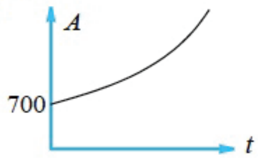


13. a $2 million b $1.589 mil c 30.1 years d

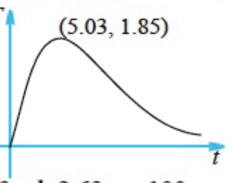


14. b 0.01761 c 199 230 d 22.6 years
⟩ a 20 cm^2 b 19.72 cm^2 c 100 days d 332 days

16. a 1 b i 512170 ii 517217 c 54.1 early 2014
17. a i $933.55 ii $935.50 b 11.95 years c



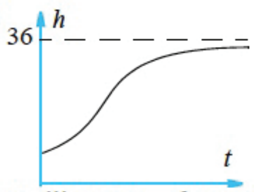
18. a 99 b $99 \times 2^{0.1394t}$ c 684
19. a



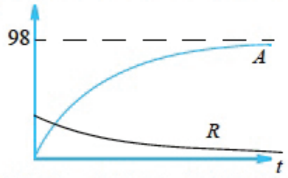
15.

b 38.85°C at ~ midnight

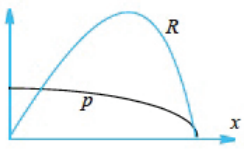
20. a 19 b 2.63 c 100
21. a 18 cm b 4 cm c 1.28 m d 36 m
e i 21.7yr ii 27.6yr iii 34.5yr f 36 g



22. a 5 mg/min b 13.51 min c i 2.1 ii 13.9 iii 68 min d 19.6 mg
e f No



23. a i $499 ii $496 iii $467 c 15537 d i $499 k ii $2.48 mil iii $4.67 mil
f 12358 g $5.14 mil b & e



Exercise B.4.5

1 **a** 5, 24, 11, 19 **b** $T = 5\sin\left(\frac{\pi t}{12} - 3\right) + 19$ **c** 23.6°

2 **a** 3, 4.2, 2, 7 **b** $L = 3\sin\left(\frac{\pi t}{2.1} - 3\right) + 7$

3 **a** 5, 11, 0, 7 **b** $V = 5\sin\left(\frac{2\pi t}{11}\right) + 7$

4 **a** 1, 11, 1, 12 **b** $P = \sin\frac{2\pi}{11}(t-1) + 12$

5 **a** 2.6, 7, 2, 6 **b** $S = 2.6\sin\frac{2\pi}{7}(t-2) + 6$

6 **a** 0.6, 3.5, 0, 11 **b** $P = 0.6\sin\left(\frac{4\pi t}{7}\right) + 11$

7 **a** 0.8, 4.6, 2.7, 11 **b** $D = 0.8\sin\frac{\pi}{2.3}(t-2.7) + 11$

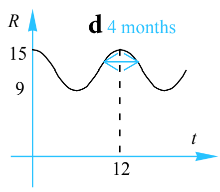
8 **a** 3000 **b** 1000, 5000 **c** $\frac{4}{9}$

9 **a** 6.5 m, 7.5 m **b** 1.58 sec, 3.42 sec

10 **a** 750, 1850 **b** 3.44 **c** mid-April to end of August

11 **a** 15000 **b** 12 months

c



d 4 months

12 **a** 2s **b** 26cm **c** 40s

d [18,34] **d** 8cm **e** 2s

f $D(t) = 8\sin\left(\pi\left(x + \frac{1}{2}\right)\right) + 26$ (for example) **g** 34cm

13 **a** $D(t) = 20\sin\left(\frac{5\pi}{6}(x + 0.2)\right) + 52$ (for example) **b** 72cm

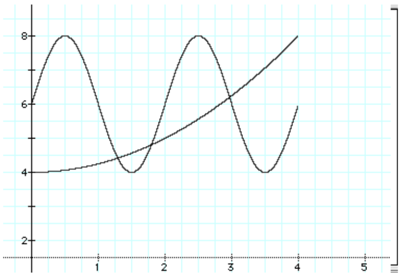
c 62cm **d** 0.86s

14 **a** π, −2, 2 **b** $\frac{1}{3}$ m **c** $\frac{4}{3}$ m

15 a

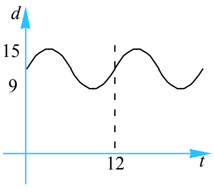
t	0	0.5	1	1.5	2	2.5	3	3.5	4
$F(t)$	6	8	6	4	6	8	6	4	6
$G(t)$	4	4.0625	4.25	4.5625	5	5.5625	6.25	7.0625	8

b



c 3 **d** 38.4%

16 **a**



b i 7,11,19,23 **ii** [0, 7] ∪ [11, 19] ∪ [23, 24] **c** 14.9 m

Exercise C.4.1

1

a $\frac{169\pi}{150}$ cm², $5.2+\frac{13\pi}{15}$ cm

b $\frac{529\pi}{32}$ cm², $23+\frac{23\pi}{8}$ cm

c 242π cm², $88 + 11\pi$ cm

d $\frac{1156\pi}{75}$ m², $13.6+\frac{68\pi}{15}$ m

e $\frac{96\pi}{625}$ cm², $1.28+\frac{12\pi}{25}$ cm

f $\frac{361\pi}{15}$ cm², $15.2+\frac{19\pi}{3}$ cm

g 5248.8π m², $648 + 32.4\pi$ cm

h $\frac{12943\pi}{300}$ cm², $17.2+\frac{301\pi}{30}$ cm

i $\frac{1922\pi}{75}$ cm², $12.4+\frac{124\pi}{15}$ cm

j $\frac{15884\pi}{3}$ cm², $152+\frac{418\pi}{3}$ cm

k 12π cm², $24 + 2\pi$ cm

l $\frac{98\pi}{3}$ cm², $28+\frac{14\pi}{3}$ cm

m $\frac{196\pi}{75}$ cm², $5.6+\frac{28\pi}{15}$ cm

n $\frac{11532\pi}{25}$ cm², $49.6+\frac{186\pi}{5}$ cm

o $\frac{3\pi}{50}$ cm², $2.4+\frac{\pi}{10}$ cm

2 0.63^c, 36°

3 0.0942 m³

4 1.64^c

5 79 cm

6 5.25 cm²

7 $\frac{\sqrt{50}\pi}{5}^c$

8 **a** 31.83 m **b** 406.28 m **c** 11°

9 1.11^c

10 0.75^c

11 **a** 1.85^c **b i** 37.09 cm **ii** 88.57 cm **c** 370.92 cm²

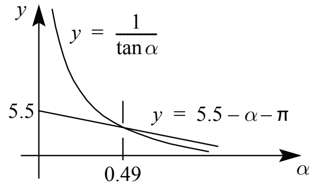
12 26.57 cm²

13 193.5 cm

14 **a** 105.22 cm **b** 118.83 cm

15 **a** 9 cm **b** 12 cm **c** 36° 52’

16 **b**



c 0.49

17 1439.16 cm^2

Exercise C.5.1

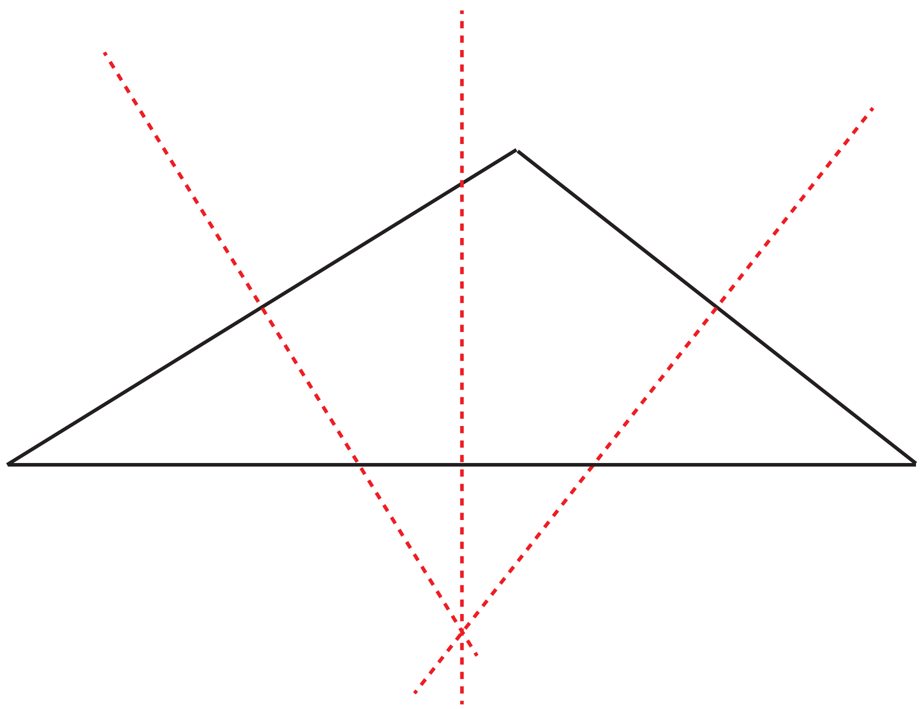
1.
a	(1,2.5)	$\sqrt{13}$	b	(2.5,2)	$\sqrt{13}$	c	(2,2.5)	$\sqrt{5}$
d	(5,5.5)	$\sqrt{9}$	e	(−1.5,−1.5)	$\sqrt{50}$	f	(−3.5,−1)	$\sqrt{17}$
g	(−1.05,1.55)	$\sqrt{112.5}$	h	(1.1,−3.2)	$\sqrt{33.8}$	i	(2.9,0)	$\sqrt{101}$

2. $9u^2$.

3. 17.374u

Exercise C.5.2

1.



2. $y = x + 2$

3. 5

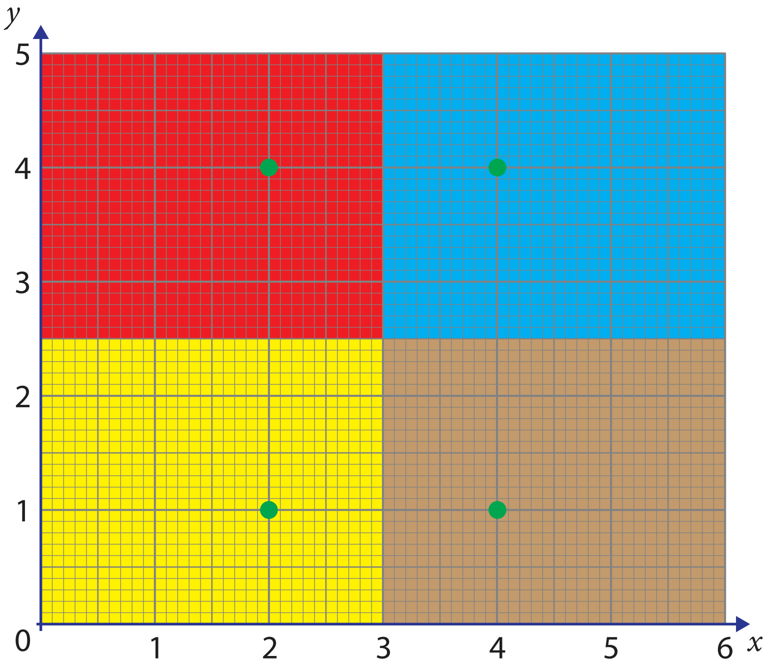
5. $(1, 2a+2)$

6. 2

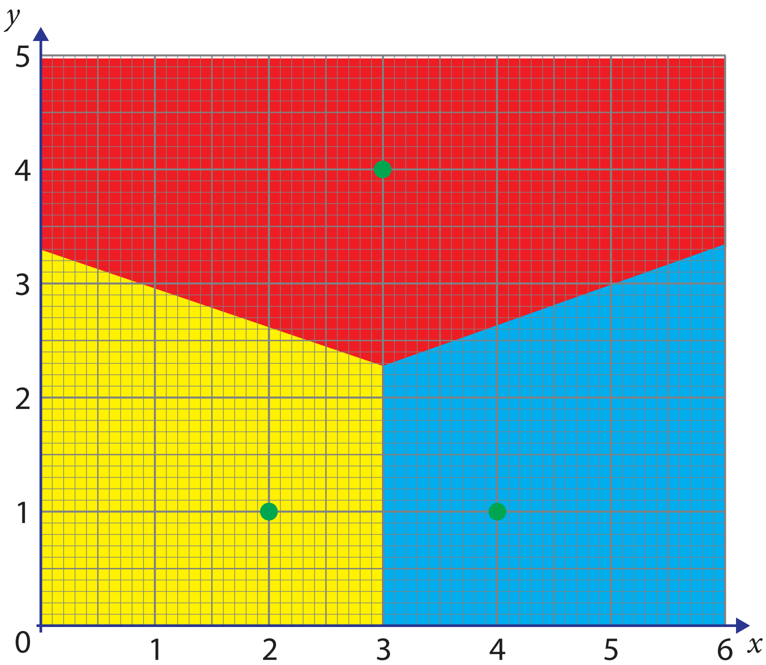
7. a Nadi (0.72,2.72) Suva (5.80,0.90) b $y = -0.3583x + 2.978$
   c $y = 2.791x - 7.289$

Exercise C.6.1

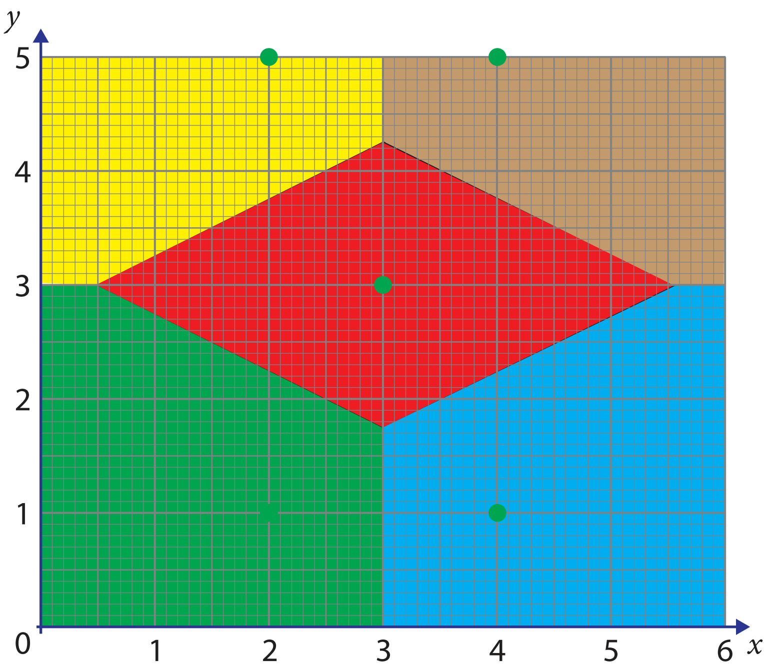
1. a



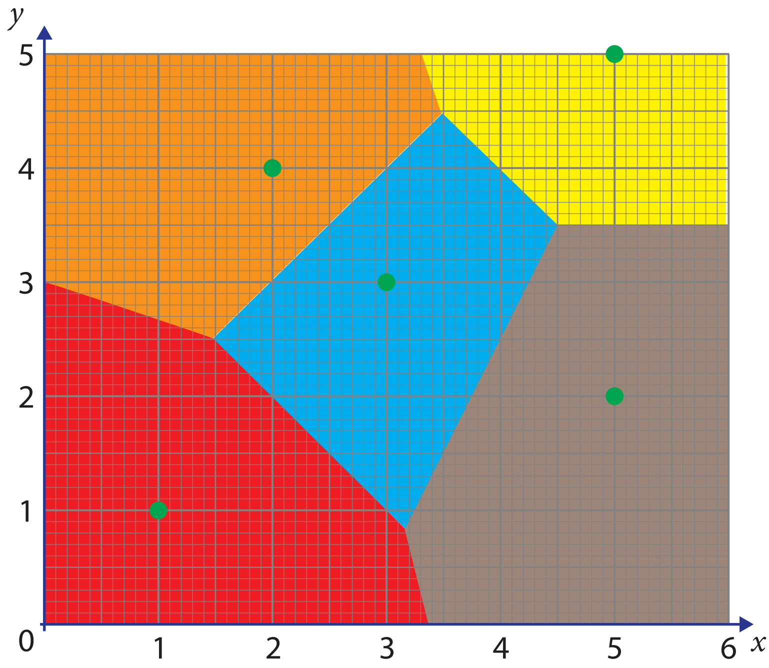
b



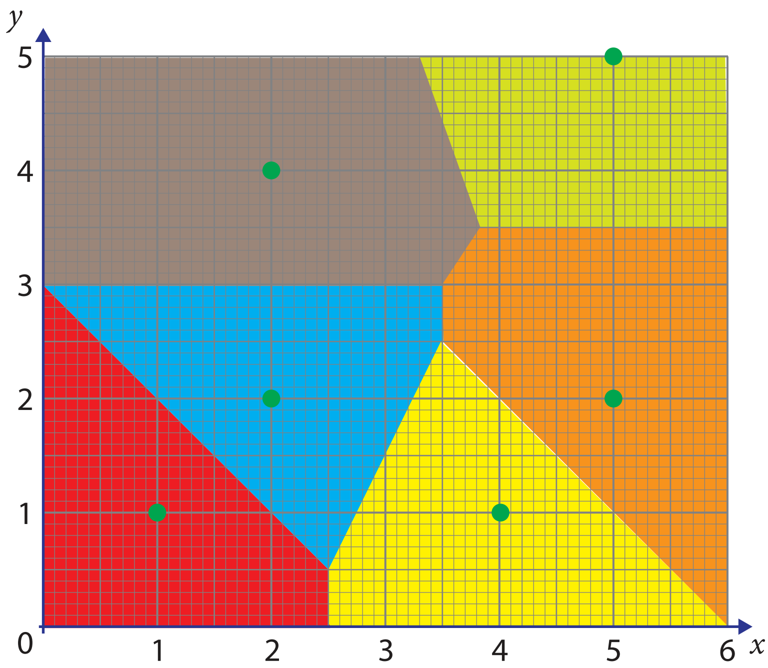
c



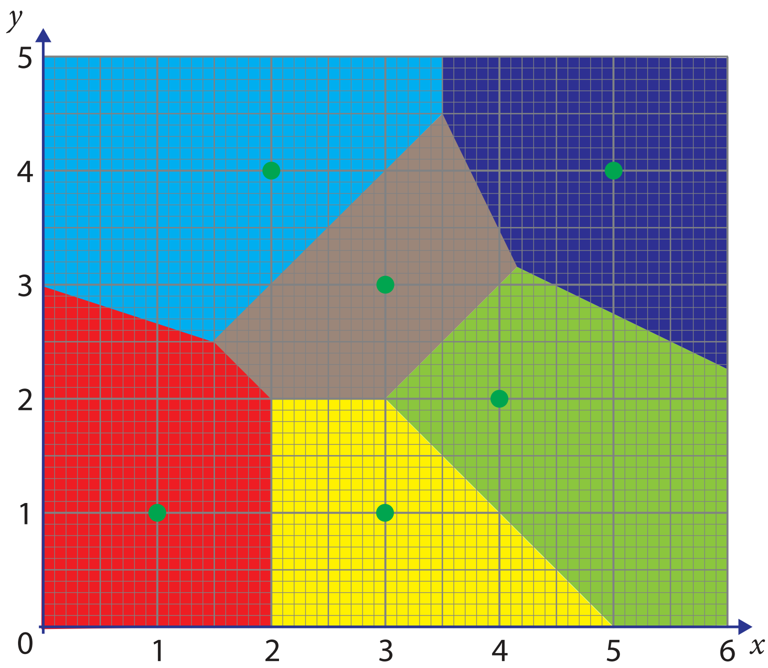
d



e



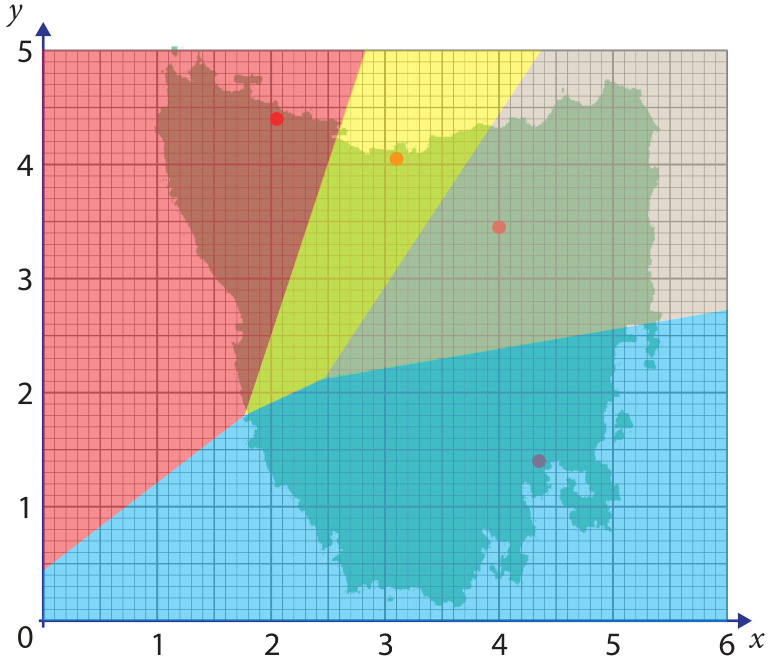
f



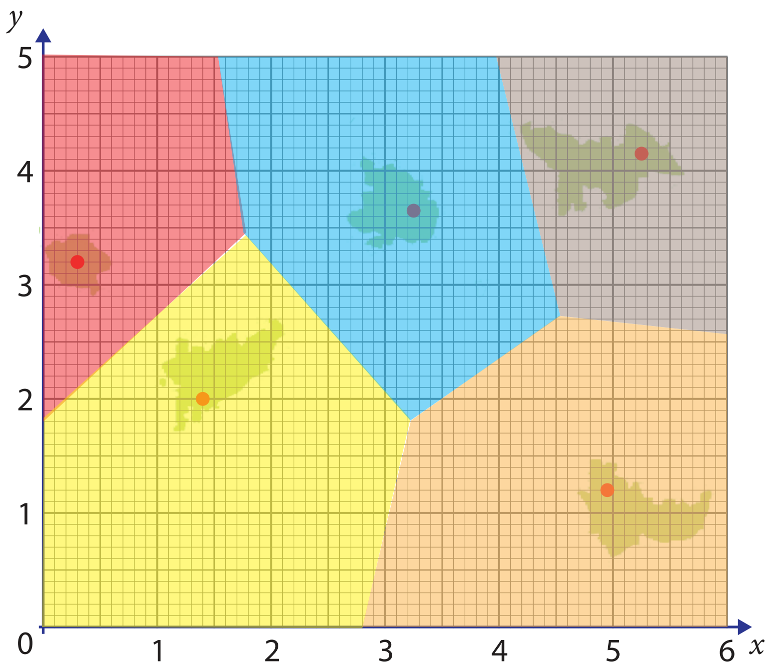
2. (3,2.5)

3. (3,2.3)

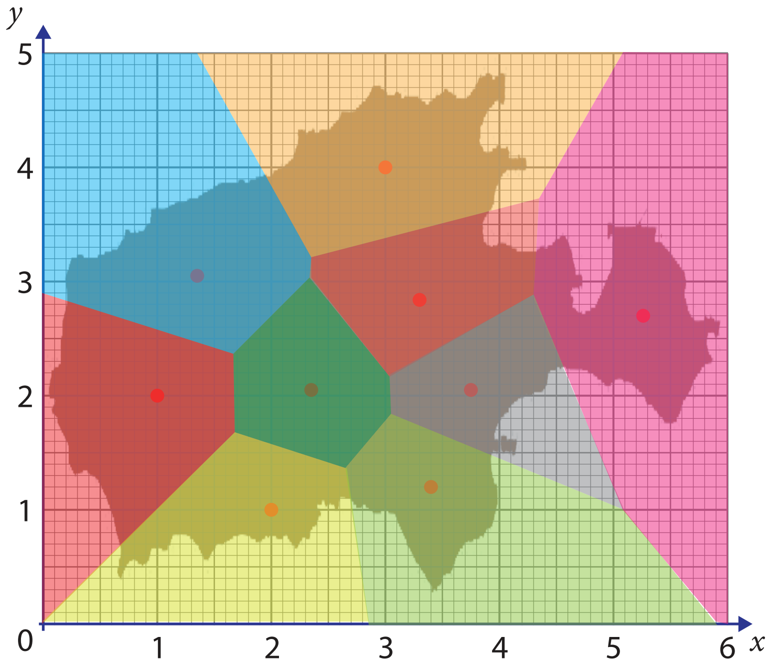
4.



5.



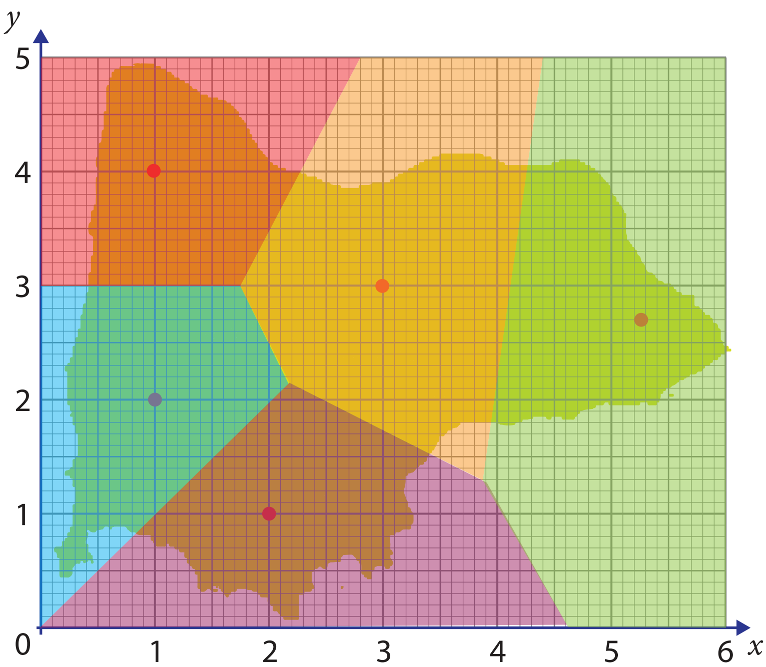
6.



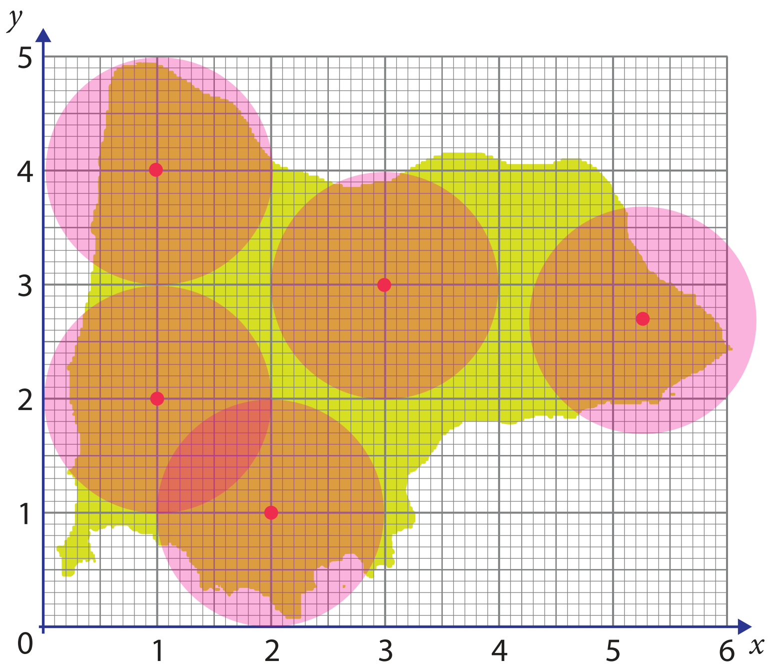
Dump at (1.75,3)

7. $\left(\frac{63}{22},\frac{45}{22}\right)$

9. a

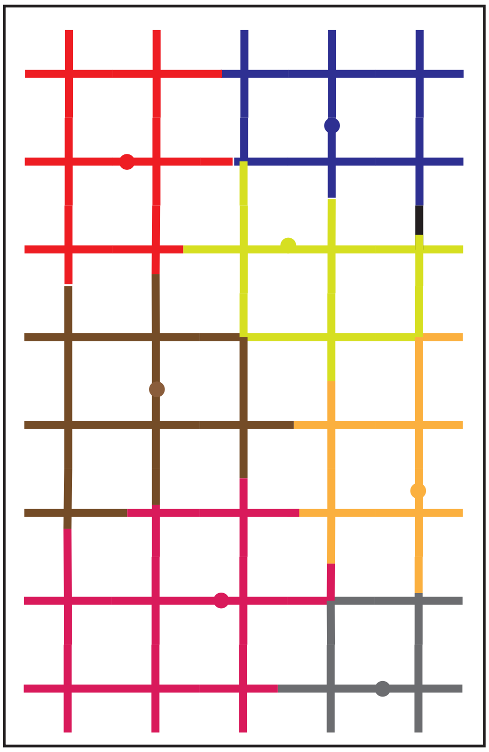


b

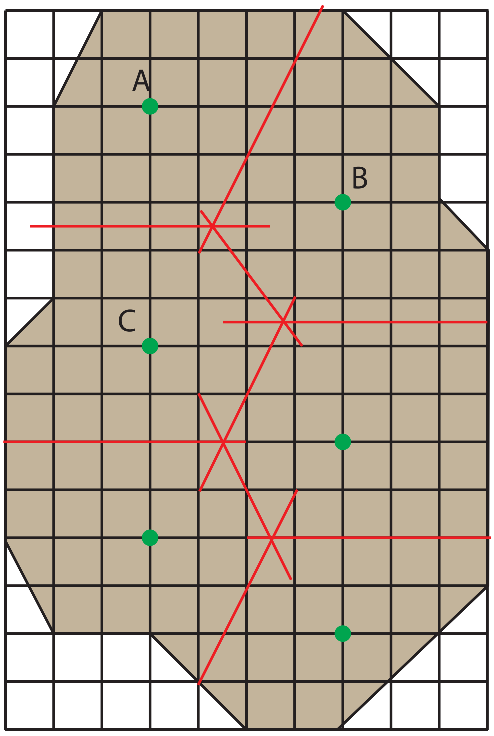


The coverage is reasonably good. Making it 100% will be difficult!

10.

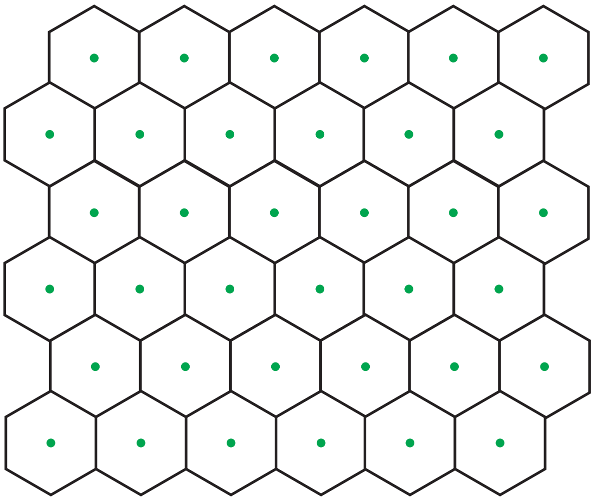


11.

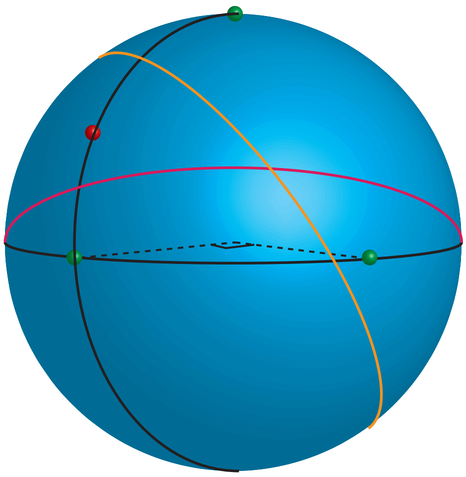


a Store A ~ 15% b Stores B & C ~35%

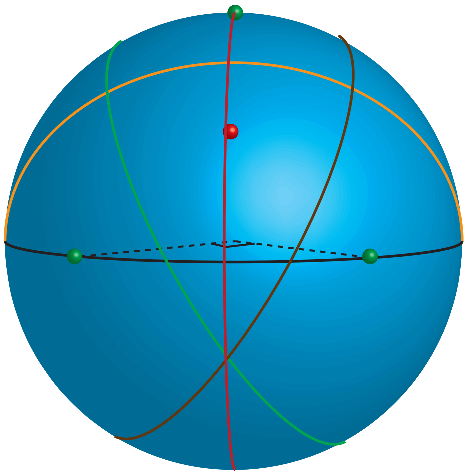
12. The Voronoi solution is the familiar 'honeycomb' of hexagonal cells.



13. a



b



c The boundaries are the arcs of great circles (centred on the centre of the sphere).

Exercise D.7.1

1. a 0.8 b 0.771 c –0.886
   d –0.143 e 0.357 f

2.

Student	Test 1	Test 2	Rank *T1*	Rank *T2*	*d*	d^2
1	3	14	15	13	2	4
2	96	97	2	2	0	0
3	61	56	5	6	-1	1
4	99	100	1	1	0	0
5	68	57	4	4	0	0
6	23	13	11	14	-3	9
7	77	81	3	3	0	0
8	36	27	9	11	-2	4
9	29	36	10	9	1	1
10	49	57	8	4	4	16
11	57	56	6	6	0	0
12	14	23	13	12	1	1
13	20	29	12	10	2	4
14	54	52	7	8	-1	1
15	8	0	14	15	-1	1

$r_s = 0.925$ high positive correlation.

3.

Student	Test 1	Test 2	Rank *T1*	Rank *T2*	*d*	d^2
1	54	33	9	2	7	49
2	30	21	11	8	3	9
3	11	35	15	1	14	196
4	57	11	8	12	-4	16
5	75	23	4	6	-2	4
6	70	24	6	5	1	1
7	88	3	2	15	-13	169
8	21	17	12	10	2	4
9	64	19	7	9	-2	4
10	49	25	10	4	6	36
11	17	14	13	11	2	4
12	17	23	13	6	7	49
13	75	29	4	3	1	1
14	79	11	3	12	-9	81
15	99	5	1	14	-13	169

$r_s = -0.414$ low negative correlation. No firm conclusion justified.

4. i Not suitable - not monotonic

   ii Suitable - a small number of outliers OK.

iii Suitable - a small number of outliers OK.

iv Not suitable - not monotonic

v Suitable

vi Suitable

5. a will show no change as the ranks are unaltered. b will show a big change as many ranks altered.

i 0.8 0.9
ii –0.8 –0.8
iii 0.8 0.8
iv 0 0
v 0.7 0.9
vi neither appropriate.

Exercise D.7.2

1. Note that these are very approximate.

	r	r_s
i	0.8	0.9
ii	–0.8	–0.8
iii	0.8	0.8
iv	0	0
v	0.7	0.9
vi	neither appropriate.	

Exercise D.8.1

1.. a H_0 : The type of policy issued is independent of the location of the field office.

H_1 : The type of policy issued is dependent on the location of the field office.

b df = 4

c [B]
...29.1 29.5 16.4...
...11.2 11.4 6.3...
...23.7 24.0 13.3...

d $\chi^2 = 9.01 < c^2 = 9.47$, so χ^2 is in the acceptance region, so we accept H_0, that the type of policy issued is independent of the location of the field office.

2. a H_0 : The level assigned is independent of gender.

H_1 : The level assigned is dependent on gender.

b df = 2

c p-value = 0.00929 < 0.05 = SL, so we accept H_1, that the level assigned is dependent on gender.

3. H_0 : The quality of the engines is independent of the day that they were produced.

H_1 : The quality of the engines depends on the day that they were produced.

Since the p-value = 3.47% < 5% = SL we accept H_1 that the quality of the engines depends on the day that they were produced.

4. H_0 : The number of credit cards possessed is independent of the age of the cardholder.

H_1 : The number of credit cards possessed depends on of the age of the cardholder.

Since the *p*-value = 12.0% > 5% = SL we accept H_0 that the number of credit cards possessed is not related to the age of the cardholder.

5. H_0 : The probability of colour-blindness is independent of gender.

H_1 : The probability of colour-blindness depends on gender.

Since the *p*-value = 13.4% > 5% = SL we accept H_0 that the probability of colour-blindness is independent of gender, i.e. the researchers' claim is not justified.

6. H_0 : The percentage of sons taking up the profession of their father is the same in every profession.

H_1 : The percentage of sons taking up the profession of their father is not the same in every profession.

Since the *p*-value = 33.4% > 5% = SL we accept H_0 that the percentage of sons taking up the profession of their father is the same in every profession.

Exercise B.8.2

1. Since the p-value = 0.565% < 1% = SL, we accept H_1 that the preferences for the five brands are not equal, i.e. not uniformly distributed

2. Since the p-value = 0.158 > 10% = SL, we accept H_0 that the distribution can be modelled by a binomial distribution with $n = 10$ and $p = 0.30$.

3. Since the p-value = 0.118 > 5% = SL, we accept H_0 that the distribution can be modelled by a Poisson probability distribution with $m = 5$.

4. Since the p-value is 20.2% > SL = 5%, we accept H_0, that the sales follow a normal distribution with average 28 and standard deviation 5.

5. Since the p-value is 6.00% > SL = 5%, we accept H_1, that accidents are not uniformly distributed by day of week.

Exercise B.8.3

1. H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \neq \mu_2$, $p = 0.0791 < 10\%$ accept H_1 that the mean weights to the tomatoes from the two patches are different.

2. H_0: $\mu_O = \mu_N$; H_1: $\mu_O > \mu_N$, $p = 0.0132 < 5\%$ accept H_1 that the new formula clears acne up faster.

3. H_0: $\mu_S = \mu_C$; H_1: $\mu_S > \mu_C$, $p = 0.126 > 10\%$ accept H_0 that the supplement was not faster.

4. H_0: $\mu_M = \mu_H$; H_1: $\mu_M \neq \mu_H$, $p = 0.0980 < 10\%$ accept H_1 that the students who did most of their homework did have higher grades than those who did half of their homework.

5. H_0: $\mu_W = \mu_M$; H_1: $\mu_W > \mu_M$, $p = 0.0248 > 1\%$ accept H_0 that men and women have the same SSHA scores.

Exam style questions

1. a 1.31 b 0.2186, c Accept H_0 accept that binomial distribution is a good fit.

2. a Reject H_0 conclude that die is fair [outcomes are uniformly distributed],

   b Accept H_0 .

3. Accept H_0, accept that the number of defects is independent of the model of car.

4. Reject H_0, reject that the weights are normally distributed with a mean of 77 kg.

5. a Accept H_0 accept that the B(12, 0.3) distribution can be used to model these results.

6. Reject H_0 Reject that B(4, 0.25) can be used to model the distribution of strawberry candies.

7. a H_1: It is not the case that one die is numbered 1, 2, 3 and 4 and the other one is numbered 1, 1, 4 & 4., (b)

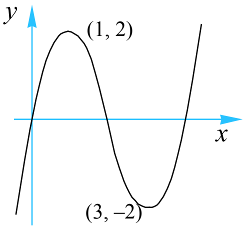
   Accept H_0.

8. a Reject H_0 and accept H_1 accept that the safety feature preferred depends on gender, (b) 5% level:

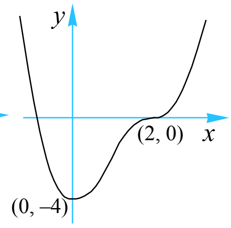
   Reject H_0. At 1% level accept H_0

Exercise E.7.1

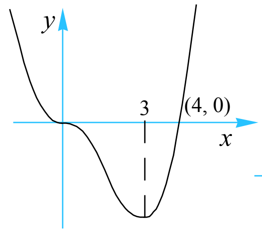
1 **a**



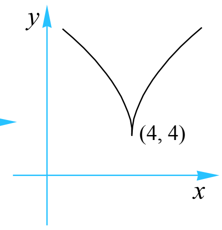
b



c



d



2 **a** max at (1, 4) **b** min at $\left(-\frac{9}{2}, -\frac{81}{4}\right)$ **c** min at (3, –45) max (–3, 63)

d max at (0, 8), min at (4, -24) **e** max at (1, 8), min at (–3, –24)

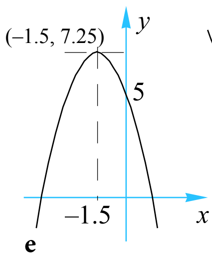
f min at $\left(\frac{1+\sqrt{13}}{3}, \frac{70-26\sqrt{13}}{27}\right)$, max at $\left(\frac{1-\sqrt{13}}{3}, \frac{70+26\sqrt{13}}{27}\right)$ **g** min at (1, –1)

h max at (0, 16), min at (2, 0), min at (–2, 0) **i** min at (1, 0) max at $\left(-\frac{1}{3}, \frac{32}{27}\right)$

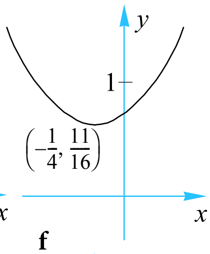
j min at $\left(\frac{4}{9}, -\frac{4}{27}\right)$ **k** min at (2, 4), max at (–2, –4)

l min at (1, 2), min at (–1, 2)

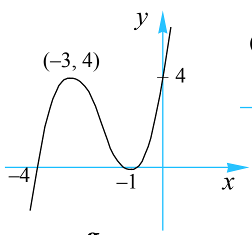
3 a



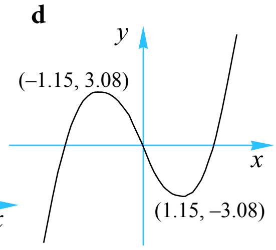
b



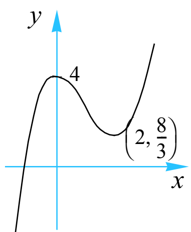
c



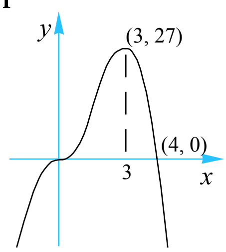
d



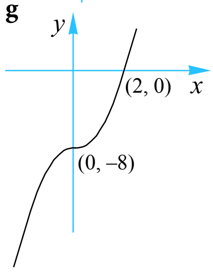
e



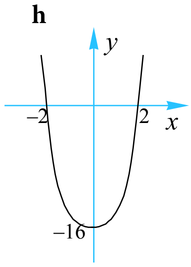
f



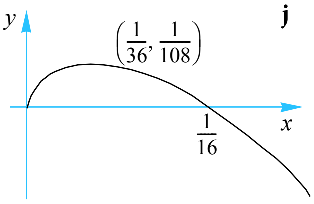
g



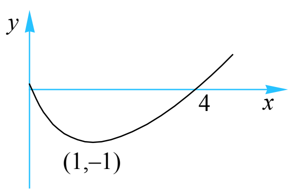
h



i

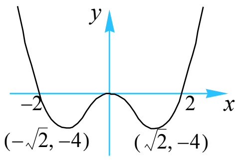


j



4 min at (1, –3), max at (–3, 29), non-stationary infl (–1, 13)

5



6. 1

7. Minimum at $\left(-\frac{\sqrt[3]{2^2}}{2}, \frac{3\sqrt[3]{2}}{2}\right)$.

8. None.

9. 8

10. –3

Exercise E.8.1

1. 22.6 m
2. a 1.5 mh–1 b $19.55 per km
3. a 400 b $46 400 000
4. $273.86
5. 0.45 m^3
6. 5 m by 5 m
7. 128
8. dim of rect. i.e. aprrox 7.00 m by 7.00 m
9. 648 m^2
10. a 10.5 b 5.25
11. 72
12. a $y = 100 - 2x$ b $A = x(100 - 2x), 0 < x < 50$ c $x = 25, y = 50$
13. a $\frac{100}{x} - \frac{1}{2}x, 0 < x < 10\sqrt{2}$ b $\frac{2000}{9}\sqrt{6} \approx 544.3 \text{ cm}^3$
14. a 400 mLs^{-1} b 40 s
15. b 8.38, 71.62 c $9 \le x \le 71$ d $80x - x^2 - 600$, $1000
16. $\left(\sqrt{\frac{11}{2}}, \frac{7}{2}\right) \& \left(-\sqrt{\frac{11}{2}}, \frac{7}{2}\right)$
17. ~ 243.7 cm^2

Exercise E.9.1

1.	a	72	b	78	c	136	d	42
	e	–206	f	9.1875	g	4.75		173.75

2. 16.5

3.	a	1.6833	b	15.8
	c	8.32		

4. 456 ot 4.56 Joules

6. 3.105

Exercise A.6.1

5. Find the solution sets of the following simultaneous equations, solving for x and y.

a
$$\begin{aligned} bx + y &= a \\ ax - y &= b \end{aligned}$$

b
$$\begin{aligned} bx + y &= a \\ ax + y &= b \end{aligned}$$

c
$$\begin{aligned} ax + by &= 1 \\ ax - by &= 1 \end{aligned}$$

d
$$\begin{aligned} ax + y &= ab \\ bx - y &= b^2 \end{aligned}$$

e
$$\begin{aligned} ax + by &= a - b \\ bx + ay &= a - b \end{aligned}$$

f
$$\begin{aligned} ax + y &= b \\ bx + ay &= 2ab - a^3 \end{aligned}$$

Exercise B.4.4

9. An endangered species of animal is placed into a game reserve. 150 such animals have been introduced into this reserve. The number of animals, $N(t)$, alive t years after being placed in this reserve is predicted by the exponential growth model $N(t) = 150 \times 1.05^t$.

   a Find the number of animals that are alive after:

      i 1 year ii 2 years iii 5 years

   b How long will it take for the population to double?

   c How long is it before there are 400 of this species in the reserve?

   d Sketch a graph depicting the population size of the herd over time. Is this a realistic model?

10. The processing of a type of mineral in a chemical solution has been found to reduce the amount of that mineral left in the solution. Using this chemical process, the amount W kg of the mineral left in the solution at time t hours is modelled by the exponential decay function $W = W_0 \times 10^{-kt}$, where W_0 kg is the original amount of mineral.

    It is found that 50 kilograms of mineral are reduced to 30 kilograms in 10 hours.

    a Write down the value of W_0.

    b Find the value of k (to 4 decimal places).

    c How much of the mineral will be in the solution after 20 hours?

    d Sketch the graph representing the amount of mineral *left* in the solution.

    e Sketch the graph representing the amount by which the mineral is *reduced*.

11. The temperatures of distant dying stars have been modelled by exponential decay functions. A distant star known to have an initial surface temperature of 15000°C, is losing heat according to the function $T = T_0 \times 10^{-0.1t}$, where T_0 °C is its present temperature, and T °C the temperature at time t (in millions of years).

    a Determine the value of T_0.

    b Find the temperature of this star in: i one million years ii 10 million years.

    c How long will it be before the star reaches a temperature that is half its original surface temperature?

    d Sketch a graph representing this situation.

12. The amount of radioactive material, Q grams, decays according to the model given by the equation $Q = 200 \times 10^{-kt}, t \geq 0$, where t is measured in years. It is known that after 40 years, the amount of radioactive material present is 50 grams.

    a Find the value of k (to 4 d.p.).

    b Find the amount of radioactive material present after 80 years.

    c What is the half life for this radioactive substance ? *The half-life is the time taken for the radioactive material to deacy to half its original amount.*

    d Sketch the graph representing the amount of radioactive material present as a function of time, t years.

13. The resale value, V dollars, of a structure, decreases according to the function

    $$V = 2000000(10)^{-0.01t}, t \geq 0$$

    where t is the number of years since the structure was built.

    a How much would the structure have sold for upon completion?

    b How much would the structure have sold for 10 years after completion?

    c How long will it take for the structure to lose half its value? (Answer to 1 d.p)

    d Sketch the graph of the structure's value since completion.

14. The population number N in a small town in northern India is approximately modelled by the equation $N = N_0 \times 10^{kt}, t \geq 0$, where N_0 is the initial population and t is the time in years since 1980.

The population was found to increase from 100 000 in 1980 to 150 000 in 1990.

a Show that $N_0 = 100000$ and that $1.5 = 10^{10k}$.

b Hence find the value of k (to 5 d.p.).

c Find the population in this town in 1997.

d How long (since 1980) will it be before the population reaches 250 000?

15. The healing process of certain types of wounds is measured by the decrease in the surface area that the wound occupies on the skin. A certain skin wound has its surface area modelled by the equation $S = 20 \times 2^{-0.01t}, t \geq 0$ where S square centimetres is the unhealed area t days after the skin received the wound.

a a What area did the wound originally cover?

b b What area will the wound occupy after 2 days?

c c How long will it be before the wound area is reduced by 50%?

d d How long will it be before the wound area is reduced by 90%?

16. In a certain city the number of inhabitants, N, at time t years since the 1 January 1970, is modelled by the equation $N = 120000(1.04)^{kt}, t \geq 0, k > 0$.

On 1 January 1980, the inhabitants numbered 177 629.

a Determine the value of k.

b How many people were living in this city by:

i 1 January 2007?

ii 1 April 2007?

b How long did it take for the population to reach 1 000 000?

17. Suppose you deposited $700 into an account that pays 5.80% interest per annum.

a How much money will you have in the account at the end of 5 years if:

i the interest is compounded quarterly?

ii the interest is compounded continuously?

b With continuous compounding, how long will it take to double your money?

c Sketch the graph showing the amount of money in the account for part b.

18. On the 1 January 1988, a number of antelopes were introduced into a wildlife reservation, free of predators. Over the years, the number of antelopes in the reservation was recorded:

Date (day/month/year)	1/1/88	1/1/90	1/6/94	1/1/98	1/6/02	1/6/04
Number of antelopes	–	120	190	260	400	485

Although the exact number of antelopes that were placed in the reserve was not available, it is thought that an exponential function would provide a good model for the number of antelopes present in the reserve.

Assume an exponential growth model of the form $N = N_0 \times 2^{kt}, t \geq 0, k > 0$, where N represents the number of antelopes present at time t years since 1/1/80, and N_0 is the initial population size of the herd, and k is a positive real constant.

a Determine the number of antelopes introduced into the reserve.

b Determine the equation that best models this situation.

c Based on this model, predict the number of antelopes that will be present in the reserve by the year 2008.

19. Betty, the mathematician, has a young baby who was recently ill with fever. Betty noticed that the baby's temperature, T, was increasing linearly, until an hour after being given a dose of penicillin. It peaked, then decreased very quickly, possibly exponentially.

Betty approximated the baby's temperature, above 37°C by the function $T(t) = t \times 0.82^t, t \geq 0$ where t refers to the time in hours after 7.00 pm.

a Sketch the graph of $T(t)$.

b Determine the maximum temperature and the time when this occurred (giving your answer correct to 2 d.p).

20. An equation of the form $N(t) = \frac{a}{1 + be^{-ct}}, t \geq 0$, where a, b and c are positive constants, represents a logistic curve. Logistic curves have been found useful when describing a population N that initially grows rapidly, but whose growth rate decreases after t reaches a certain value.

A study of the growth of protozoa was found to display these characteristics. It was found that the population was well described if $c = 1.12$, $a = 100$, and t measured time in days.

a If the initial population was 5 protozoa, find the value of b.

b It was found that the growth rate was at a maximum when the population size reached 50. How long did it take for this to occur?

c Determine the optimum population size for the protozoa.

21. The height of some particular types of trees can be approximately modelled by the logistic function $h = \frac{36}{1 + 200e^{-0.2t}}, t \geq 0$ where h is the height of the tree measured in metres and t the age of the tree (in years) since it was planted.

a Determine the height of the tree when planted.

b By how much will the tree have grown in the first year ?

c How tall will the tree be after 10 years ?

d How tall will it be after 100 years ?

e How long will it take for the tree to grow to a height of:

i 10 metres ? ii 20 metres ? iii 30 metres ?

f What is the maximum height that a tree, whose height is modelled by this equation, will reach? Explain your answer.

g Sketch a graph representing the height of trees against time for trees whose height can be modelled by the above function.

22. Certain prescription drugs, e.g. tablets that are taken orally, which enter the bloodstream at a rate R, are approximately modelled by the equation $R = a \times b^t, t \geq 0$ where t is measured in minutes and a and b are appropriate constants.

When an adult is administered a 100-milligram tablet, the rate is modelled by the function $R = 5 \times 0.95^t, t \geq 0$ mg/min.

The amount A mg of the drug in the bloodstream at time t minutes can then be approximated by a second function, $A = 98(1 - 0.95^t)$ mg.

a What is the initial rate at which the drug enters the bloodstream?

b How long will it take before the rate at which the drug enters the bloodstream is halved?

c How long does it takes for:

i 10 milligrams of the drug to enter the bloodstream.

ii 50 milligrams of the drug to enter the bloodstream.

iii 95 milligrams of the drug to enter the bloodstream.

d How much of the drug is in the bloodstream when the drug is entering at a rate of 4 mg/min.

e Sketch the graph of R and A, on the same set of axes.

f Will the patient ever feel the full effects of the 100-milligram drug?

23. As consumers, we know from experience that the demand for a product tends to decrease as the price increases. This type of information can be represented by a demand function. The demand function for a particular product is given by $p = 500 - 0.6 \times e^{0.0004x}$, where p is the price per unit and x is the total demand in number of units.

a Find the price p to the nearest dollar for a demand of:

i 1000 units ii 5000 units iii 10 000 units.

b Sketch the graph of this demand function.

c The total revenue, R, obtained by selling x units of the product is given by $R = xp$. What level of demand will produce a price per unit of $200?

d Find the revenue by selling:

i 1000 units ii 5000 units iii 10 000 units.

e Sketch the graph of the revenue equation.

f Find the number of units that must be sold in order to maximize the total revenue.

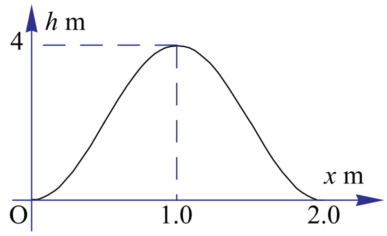
g Determine the maximum revenue, giving your answer to 2 d.p.

Exercise B.4.5

14. A hill has its cross-section modelled by the function,

$$h : [0, 2] \mapsto \mathbb{R},\ h(x) = a + b\cos(kx),$$

where $h(x)$ measures the height of the hill relative to the horizontal distance x m from O.



a Determine the values of

i k

ii b

iii a

b How far, horizontally from O, would an ant climbing this hill from O be, when it first reaches a height of 1 metre?

c How much further, horizontally, will the ant have travelled when it reaches the same height of 1 metre once over the hill and on its way down?

15. A nursery has been infested by two insect pests: the Fruitfly and the Greatfly. These insects appear at about the same time that a particular plant starts to flower. The number of Fruitfly (in thousands), t weeks after flowering has started is modelled by the function

$$F(t) = 6 + 2\sin(\pi t),\ 0 \le t \le 4$$

Whereas the number of Greatfly (in thousands), t weeks after flowering has started is modelled by the function

$$G(t) = 0.25t^2 + 4,\ 0 \le t \le 4$$

a Copy and complete the following table of values, giving your answers correct to the nearest hundred.

t	0	0.5	1	1.5	2	2.5	3	3.5	4
$F(t)$									
$G(t)$									

b On the same set of axes **draw** the graphs of:

i $F(t) = 6 + 2\sin(\pi t),\ 0 \le t \le 4$. ii $G(t) = 0.25t^2 + 4,\ 0 \le t \le 4$.

c On how many occasions will there be equal numbers of each insect?

d For what percentage of the time will there be more Greatflies than Fruitflies?

16. The depth, $d(t)$ metres, of water at the entrance to a harbour at t hours after midnight on a particular day is given by

$$d(t) = 12 + 3\sin\left(\frac{\pi}{6}t\right),\ 0 \le t \le 24$$

a Sketch the graph of $d(t)$ for $0 \le t \le 24$.

b For what values of t will: i $d(t) = 10.5$, $0 \le t \le 24$ ii $d(t) \ge 10.5$, $0 \le t \le 24$.

Boats requiring a minimum depth of b metres are only permitted to enter the harbour when the depth of water at the entrance of the harbour is at least b metres for a continuous period of one hour.

c Find the largest value of b, correct to two decimal place, which satisfies this condition.

Exercise C.4.1

1. Find the areas and perimeters of the following sectors.

	Radius	Angle
h	8.6 cm	$\frac{7\pi}{6}$
i	6.2 cm	$\frac{4\pi}{3}$
j	76 m	$\frac{11\pi}{6}$
k	12 cm	30°
l	14 m	60°
m	2.8 cm	120°
n	24.8 cm	270°
o	1.2 cm	15°

14. Two pulleys of radii 7 cm and 11 cm have their centres 24 cm apart. Find the length of the piece of string that will be required to pass tightly round the circles if:

a the string cannot cross over.

b the string crosses over itself.

15. A sector of a circle has a radius of 15 cm and an angle of 216°. The sector is folded in such a way that it forms a cone, so that the two straight edges of the sector do not overlap.

a Find the base radius of the cone.

b Find the vertical height of the cone.

c Find the semi–vertical angle of the cone.

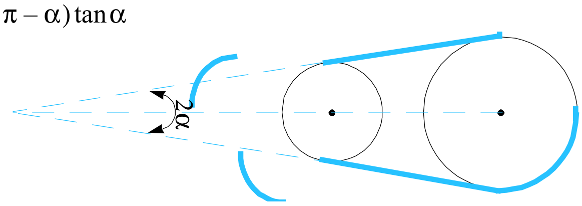
16. A taut belt passes over two discs of radii 4 cm and 12 cm as shown in the diagram.

a If the total length of the belt is 88 cm, show that $1 = (5.5 - \pi - \alpha)\tan\alpha$

b On the same set of axes, sketch the graphs of:

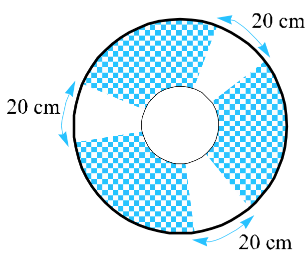
i $y = \frac{1}{\tan\alpha}$

ii $y = 5.5 - \pi - \alpha \cdot$

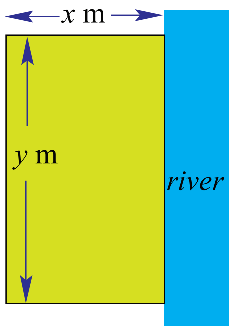


c Hence find $\{\alpha : 1 = (5.5 - \pi - \alpha)\tan\alpha\}$, giving your answer to two d.p.

17. The diagram shows a disc of radius 40 cm with parts of it painted. The smaller circle (having the same centre as the disc) has a radius of 10 cm. What area of the disc has not been painted in blue?



Exercise E.8.1



12. A farmer wishes to fence off a rectangular paddock using an existing stretch of a river as one side. The total length of wiring available is 100 m.

Let x m and y m denote the length and width of this rectangular paddock respectively, and let $A\ \text{m}^2$ denote its area.

a Obtain an expression for y in terms of x.

b Find an expression for A in terms of x, stating any restrictions on x.

c Determine the dimensions which will maximize the area of the paddock.

13. A closed rectangular box with square ends is to be constructed in such a way that its total surface area is 400 cm². Let x

14. A barrel is being filled with water in such a way that the volume of water, V mL, in the barrel after time t seconds is given by

$$V(t) = \frac{2}{3}\left(20t^2 - \frac{1}{6}t^3\right),\ 0 \le t \le 120\ .$$

a Find the rate of flow into the barrel after 20 seconds.

b When will the rate of flow be greatest?

c Sketch the graph of $V(t),\ 0 \le t \le 120$ .cm be the side length of the ends and y cm its height.

a Obtain an expression for y in terms of x, stating any restrictions on x.

b Find the largest possible volume of all such boxes.

15. The total cost, \$$C$, for the production of x items of a particular product is given by the linear relation $C = 600 + 20x,\ 0 \le x \le 100$, whilst its total revenue, \$$R$, is given by $R = x(100 - x),\ 0 \le x \le 100$.

a Sketch the graphs of the cost function and revenue function on the same set of axes.

b Determine the break-even points on your graph.

c For what values of x will the company be making a positive profit?

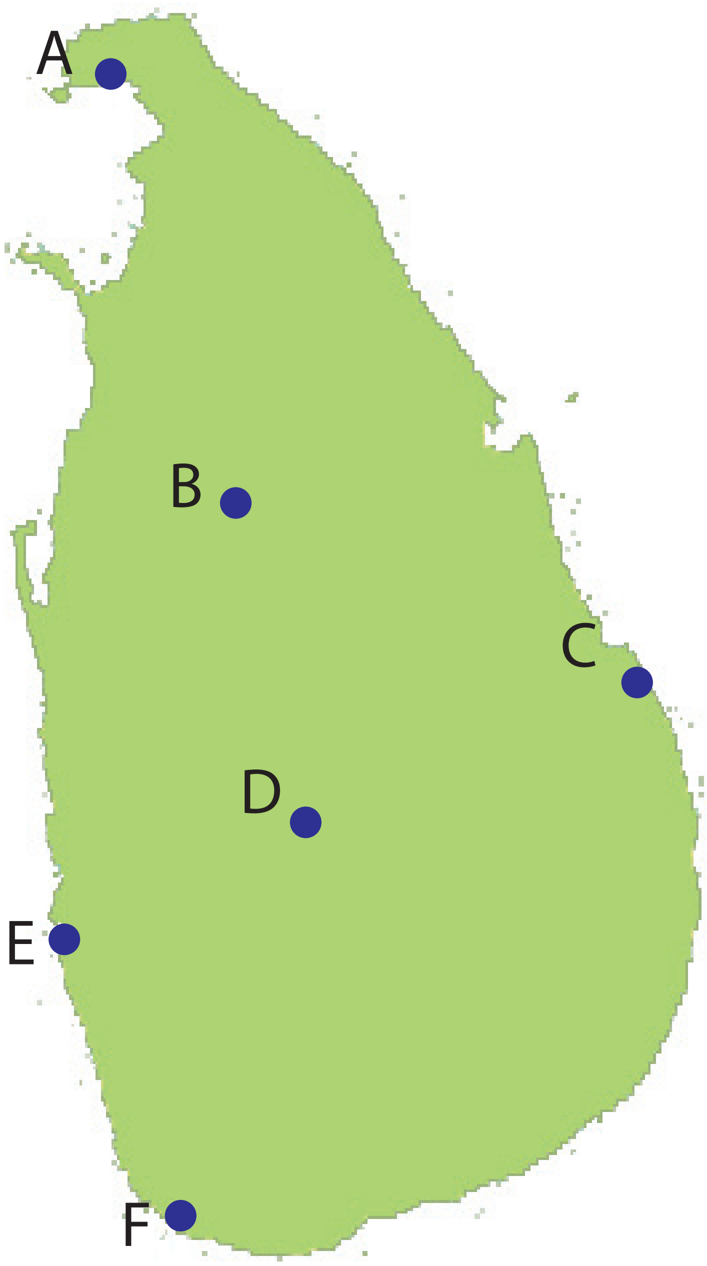
d Find an expression that gives the profit made in producing x items of the product and hence determine the maximum profit.

16. Find the points on the graph of $y = 9 - x^2$ that are closest to the point (0, 3).

17. A certificate is to be printed on a page having an area of 340 cm². The margins at the top and bottom of the page are to be 2 cm and, on the sides, 1 cm.

a If the width of the page is x cm, show that the area, A cm² where printed material is to appear is given by $A = 348 - \frac{680}{x} - 4x$

b Hence, determine the maximum area of print.


A
B
C
D
E
F

Applications SL Errata

Applications SL Supplement